# STRATEGY USE INDICATIVE OF AN UNDERSTANDING OF UNITS OF LENGTH

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*The purpose of this report is to compare the effects of presentation mode based on students' strategy use in measurement tasks. The same treatment was presented, either with Geometer's Sketchpad (GSP) images and animations or within a paper-and-pencil environment to 36 students (21 aged 4-5 years, and 15 aged 7-8 years. Changes in student strategy use were tracked by following a microgenetic method over 18 trials. The non-computer and computer groups experienced similar shifts in strategy and both were substantially different from the shifts experienced by the comparison group. Additionally, the strategies which emerged during this study were compared based on the percentage of times they were used to produce correct responses. Through this analysis, three highly effective strategies emerged.*

## INTRODUCTION

An understanding of linear measure is imperative, as it provides the basis for length, area, and volume. In each of the instances, students depend on one tool, the ruler, to gather the information needed to calculate the measure required. Many design decisions have been made so that rulers can be used easily and effectively; however, as a result of imbedding these decisions in the ruler's design, many students' misunderstandings are potentially hidden from teachers (Lehrer & Schauble, 2000).

Researchers have noted that the difficulties students have with unpacking the meaning encapsulated in the design of a ruler may stem from their inability to abstract how the tool displays a linear unit. Cannon (1992) reflected on the fact that the "use of a ruler reinforces the notion that there is a direct relationship between the count of points and the number of units" (p. 111). Bragg and Outhred (2004) stated that when students are "asked to indicate which feature of the scale on a ruler is counted when measuring a length" (p. 160) many are likely to indicate the hash marks on the tool rather than the line segments defined by these marks.

Research prior to this study identified three different possible interventions to promote conceptual knowledge of units of length. The first is to emphasize a sweeping motion through an interval to identify the units of length along a measurement tool. This motion is meant to help students see the unit markers "as the point where each unit starts and ends" (Bragg & Outhred, 2004, p.164). The second suggestion is to have students develop an understanding of length measurement through the process of building segments by iterating smaller unit segments (Barrett & Clements 2003; Kamii, 1995). The final suggestion is implied by Barrett et al. (2009) in which they discuss the transition from a tool built from individual 1-inch



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yellow strips to a standard ruler. In this study the researchers found that students were very successful when using a tool built from 12 individual 1-inch yellow strips of paper. This tool was used successfully to measure the length of objects, even when the object was not aligned to one of the endpoints of the tool. The researchers reported their own surprise when a student was unable to transfer this skill from the yellow strips tool to another tool resembling a standard ruler with no numerals. The present study picked up where this intervention technique left off and set out to explore more direct ways to display how a ruler encapsulates the laying of units end to end in a succession to quantify the length of an object.

Lessons on measurement require attention to representational media, with technology, and computer technology, in particular, becoming important parts of the modern classroom. Computer technology provides teachers with ways of illustrating abstract ideas and ways of assessing students' thinking. This fact makes it imperative that we "understand how and why technologies influence, and will continue to influence, mathematics education" (Mariotti, 2002, p. 695). Some of the most widely used technologies are dynamic geometry environments (DGEs) (Becker, 2000). Although many mathematics teachers as well as many mathematics education researchers focus their attention on the use of DGEs (McClintock, Zhonghong, & Raquel, 2002), there are aspects of this realm that need further investigation. One such example, according to Battista (2007), is that although we know that "DGEs encourage and support significant learning in students" what seems to be missing is how "learning in DGEs differs from learning in paper-and-pencil environments. Thus we need more comparison studies, both quantitative, to investigate generality, and qualitative, to investigate differences in cognitive processes" (p. 904).

It seems that investigating students' development of geometric measurement concepts, specifically the coordination of the markings on a ruler and the iteration of a unit of length would be time well spent. Several instructional approaches might be expected to support this type of conceptual integration between iterative units along an object and the markings one finds on rulers.

The purpose of this study was to explore students' shifting strategies as they learned to coordinate the markings on a ruler to the iteration of a unit of length. This study also compared the effectiveness of presenting the tasks using one of two visual representations, either with Geometer's Sketchpad (GSP) images and animation or within a paper-and-pencil environment.

## RESEARCH QUESTIONS

Students' strategy use was monitored over 18 trials to examine shifts in strategy use by two treatment groups and a comparison group. In particular, this study addressed two questions: (1) how do shifts in students' strategy use differ based on mode of treatment presentation, either a computer environment or a paper-and-pencil environment? (2) Which strategies, if any, are more likely to lead students to develop an understanding of how a ruler portrays units of length?

## THEORETICAL FRAMEWORK

We hope to understand how students change from less effective to more effective reasoning and strategies for measuring. If and when positive changes are brought about in a student's thinking habits it is important that we are able to identify and isolate what chain of events brought about these changes. The accumulation of these changes is what leads students to develop. "However, determining how change occurs is very difficult" (Siegler & Crowley, 1991, p. 606). An attempt to develop a method to study change in a systematic manner has resulted in what is most commonly known as the microgenetic method. According to Siegler and Svetina (2006) the microgenetic method has three main properties:

> (1) observations span the whole period of rapidly changing competence; (2) the density of observation within this period is high, relative to the rate of change; and (3) observations of changing performance are analyzed intensively to indicate the processes that give rise to them (p. 1000).

To cope with the need for dense sampling over what may be a relatively long time period researchers using the microgenetic method have developed two distinct strategies. The first is to identify a task from the everyday environment that is likely to support change and to "provide higher concentration of such experiences than would otherwise occur" (p. 607). The second approach is to provide children with a novel task and to observe their "changing understanding of it within a single session or over multiple sessions" (p. 607). This study employed the second approach by providing students tasks which they are unlikely to encounter outside of such a research project.

## METHODOLOGY

This study was a comparative analysis of two different treatment presentation modes designed to prompt students' schemes for enumeration on a ruler and iterating a unit of length. The first mode of presentation employed the visual display capabilities of GSP while the second mimicked the visual display with concrete manipulatives. Because an instructional goal of the study is to develop a visual display that can be delivered to an entire class of young students from an overhead display, the GSP treatment was developed and administered in a manner that would not require the students to interact with the GSP presentation. A comparison group was also included in the study so that each of the presentation modes could be compared to a group of students receiving no visual support.

### Student Selection

The participants for this study were selected from a suburban school in the Midwestern portion of the United States of America. We only selected students who were identified by poor performance on measurement tasks in which the object to be measured was not aligned with the endpoint of the ruler. Students from two Kindergarten and two second-grade classrooms were assigned to one of three groups following a stratified random sample to mix grade level and classroom.

## Data Collection

For each of 18 trials (3 trials on each of six sessions spanning six weeks) the student's strategy use was tracked in relation to their performance on the measure question for each trial. In total, eight different strategies for identifying units of length were observed and used to classify student responses. The strategy code, *ep*, indicated that a student referred to either the right or left *endpoint* as the length of the object. For example, if the object was stretched from the four to the seven the student would report either four or seven as the object's measure. The second strategy (ptmid) indicated a student *pointed to the middle of an interval* defined by the two consecutive tick marks. The third strategy (ct) meant a student *counted the tick* marks. The fourth strategy (trans) described students who *translated the object* in an attempt to mentally compensate for the fact that the object was not aligned properly. The fifth strategy (swp) indicated students who *swept their finger* from one tick mark to the next and counted these sweeping motions. The sixth strategy (spanf) was used to describe what happened when students *spanned their finger* from one tick mark to the next and counted each of these spanning actions as a unit. The seventh strategy (arith) was used when the student *used arithmetic* in an attempt to calculate the length of the object being measured. We also noted if a student's strategy could not be observed or when the student was unwilling to describe his or her thinking.

## Data Analysis

For each trial question (with three trials per session), the student's response was recorded as correct or incorrect as well as their strategy for identifying the units of length. After a student reported a length on each trial, they were asked, "Can you tell me how you thought about that?" as a prompt to identify their strategy. The eight strategies listed above emerged from these observations and from our survey of the literature. In the event that a student used more than one strategy for a trial each of the relevant strategies was recorded. The strategy use and correctness data were compiled for each of the three groups across the six sessions. A graphic representation of the total number of instances of each strategy use was generated using Excel. These graphical representations provided a basis for a visual comparison between the three groups of the overall shifts in strategy use throughout the study. This provided evidence of the similarities and differences in the effects of the two treatments as well as evidence of the similarities and differences between the two treatment groups and the comparison group.

Once the correctness of responses and the strategies used for each response were collected we examined the general effectiveness of each of the strategies. The total number of times each strategy was used allowed us to compare the strategy popularity within each of the three groups. Similarly, the effectiveness of each of the strategies could be examined overall and compared from one group to the next. This allowed conclusions to be drawn about the treatment impact on strategy selection as well as conclusions about possible effective strategies for identifying units of length.

## RESULTS AND DISCUSSION

Results for the three experimental groups are shown in Figures 1, 2 and 3, which display the patterns of strategy use throughout the study.

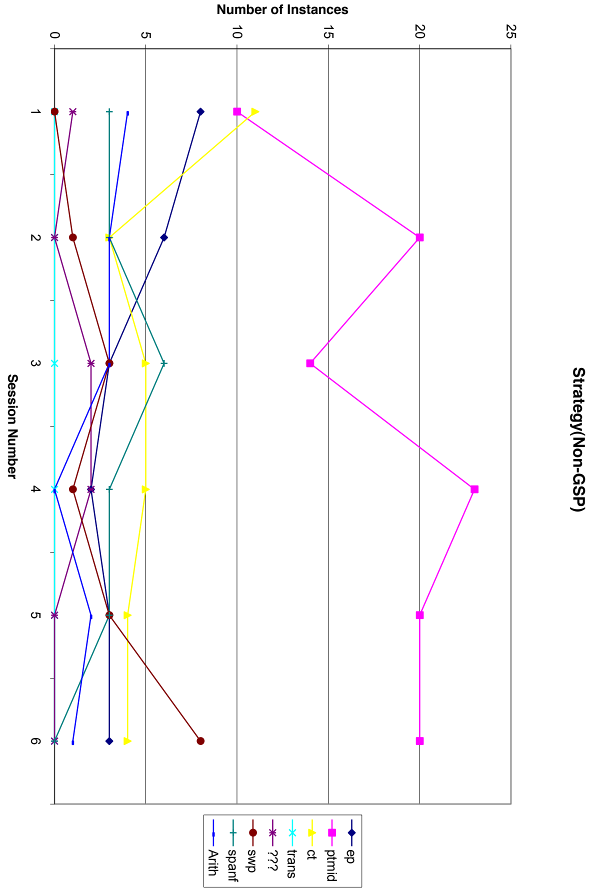


*Figure 1.* Non-GSP strategy use by session.

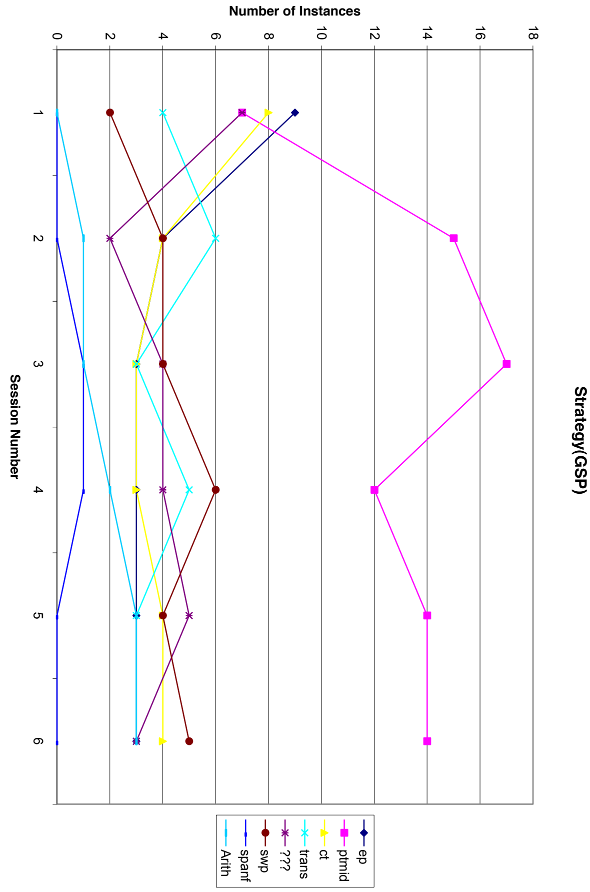


*Figure 2.* GSP strategy use by session.

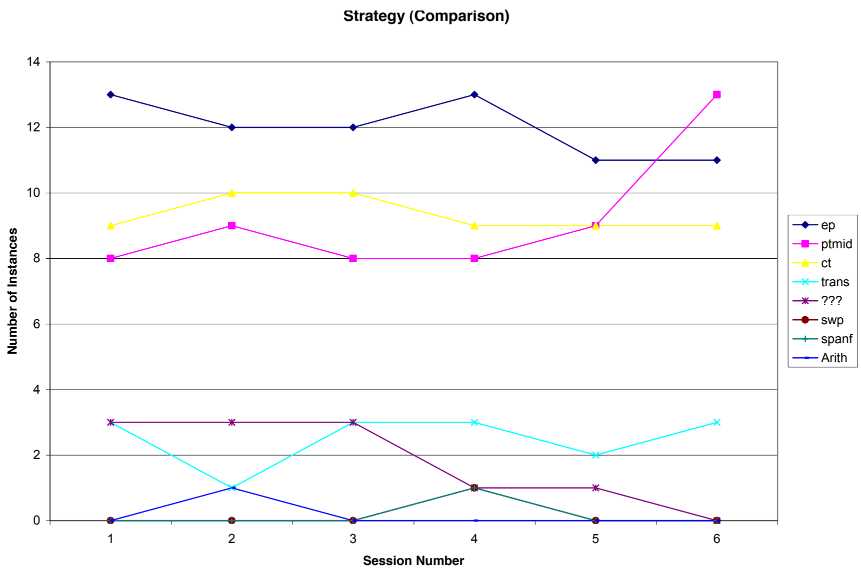


*Figure 3.* Comparison strategy use by session.

We compiled the total number of times each strategy was used during each weekly session. On this basis, we compared the effects of the different treatments used throughout this study. Then we explored the percentage of correct responses produced by each strategy.

### Shifts in Strategy Use

Analysing strategy use by session for each of the three groups revealed strong similarities between the GSP and Non-GSP group and sharp contrast from these two groups to the comparison group. In all three groups, the endpoint strategy (ep), the count tick marks strategy (ct), and the point to midpoint strategy (ptmid) were used most frequently during the first session. In the GSP and Non-GSP groups, all strategies other than the point to midpoint (ptmid) strategy were suppressed by the treatments, with the point to midpoint strategy (ptmid) being used more than twice as often as any other strategy after the first session. In contrast, the comparison group continued to use the endpoint strategy (ep), the count tick marks strategy (ct), and the point to midpoint strategy (ptmid) consistently throughout all six sessions.

### Strategy Totals and Effectiveness

Next, we discuss the frequency of strategies in relation to the success of the strategies. Results indicate that three strategies, point to midpoint strategy (ptmid), sweeping the interval strategy (swp) and spanning with fingers strategy (spanf) were highly effective, but the endpoint strategy (ep) was highly ineffective (see Table 1).

Table 1

*Strategy Use and Success*

| | ptmid | ct | ep | tran | swp | arith | spanf |
|---|---|---|---|---|---|---|---|
| # of times Used | | | | | | | |
| GSP | 106 | 33 | 25 | 0 | 13 | 16 | 22 |
| Non-GSP | 80 | 28 | 25 | 24 | 25 | 10 | 2 |
| Comparison | 55 | 61 | 72 | 18 | 3 | 1 | 2 |
| Total | 241 | 122 | 122 | 42 | 41 | 27 | 26 |
| % Correct | | | | | | | |
| GSP | 99.06% | 9.09% | 0% | NA | 100% | 75% | 95.45% |
| Non-GSP | 98.75% | 64.29% | 0% | 45.83% | 100% | 100% | 100% |
| Comparison | 98.18% | 27.87% | 0% | 16.67% | 100% | 0% | 100% |
| Total | 98.76% | 31.15% | 0% | 33.33% | 100% | 81.48% | 96.15% |

The effective strategies are consistent with research in three different realms. First, although students in this study did not actively build segments by iterating smaller unit segments (Barrett & Clements, 2003; Kamii, 1995), they benefited from observing this process both in the physical and virtual environments. Second, the suggestion of using a sweeping motion through an interval to identify the units of length along a measurement tool (Bragg & Outhred, 2004) was verified. In fact even without providing support or motivation for students to use this strategy, it emerged on its own and was used with 100% (41 out of 41 times) accuracy.

The transition from the yellow strip tool to the standard ruler, as suggested in a previous study (Barrett et al., 2009), was the most influential in developing the treatment used for this study. This study found that linking the intervals on a ruler to iterable discrete objects, or to virtual representations of those objects, were both successful ways to motivate students to use the effective *point to midpoint* strategy. Based on the fact that it was used correctly over 98% of the time throughout the study, the use of this strategy appears to indicate the student has coordinated the markings on the ruler and the units of length.

## Implications for Teaching

What seems to be common among the three effective strategies is that they encourage modelling of the individual unit segments along the object. We feel that the use of any of these strategies is an indicator that the student has developed a meaningful understanding of how the units of length are portrayed on the ruler. For this reason we suggest that these interval identifying strategies be used when teaching students to measure the length of an object with a ruler.

## References

Barrett, J. E., & Clements, D. H., (2003). Quantifying length: Fourth-grade children's developing abstractions for measures of linear quantity. *Cognition and Instruction, 21*(4), 475-520.

Barrett, J. E., Clements, D. H., Cullen, C., McCool, J., Witkowski, C., & Klanderman, D. (2009, April). *Children's abstraction of iterative units to measure linear space: A trajectory.* Paper presented at the Annual Meeting of the American Educational Research Association.

Battista, M. T. (2007). The development of geometric and spatial thinking. In F.K. Lester (Ed). *Second handbook of research on mathematics teaching and learning* (pp. 843-908). Charlotte, NC. Information Age Publishing.

Becker, H. (2000, July). *Findings from the teaching, learning, and computing survey: Is Larry Cuban right?* Paper presented at the 2000 School Technology Leadership Conference of Council of Chief State School Officers, Washington, D. C.

Bragg, P., & Outhred, L. (2004). A measure of rulers–The importance of units in a measure. In M. Hoines & A. Fuglestad (Eds.), *Proceedings of the 28th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 159-166). Bergen, Norway: International Group for the Psychology of Mathematics Education.

Cannon, L., (1992). Middle grade students' representations of linear units. In W. Geeslin & K. Graham (Eds.), *Proceedings of the Sixteenth PME Conference* (Vol. I, pp. 105-112). Durham, NH: International Group for the Psychology of Mathematics Education.

Kamii, C. (1995, October). *Why is the use of a ruler so hard?* In Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Columbus, OH.

Lehrer, R., & Schauble, L. (2000). Developing model-based reasoning in mathematics and science. *Journal of Applied Developmental Psychology*, *21,* 39-48.

Mariotti, M. A. (2002). The influence of technological advances on students' mathematical learning. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 695-723). Manua, NJ: National Council of Teachers of Mathematics.

McClintock, E., Zhonghong, J., & Raquel J. (2002). Student's development of three-dimensional visualization in the Geometer's Sketchpad environment. In Proceedings of the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Athens, GA.

Siegler, R. S., & Crowley, K. (1991). The microgenetic method: A direct means for studying cognitive development. *American Psychologist, 46*, 606-620.

Siegler, R. S., & Svetina, M. (2006). What leads children to adopt new strategies? A microgenetic/cross-sectional study of class inclusion. *Child Development, 77,* 997-1015.