Three questions about developing and using LTs

1. Explain how you conduct validation studies of your LT; Explain how differing conditions of practice are handled in field-testing and implementation.

2. What do you see as the development cycle for these theoretical objects, the LTs? What is hypothetical about LTs? Do you have a process for anticipating and accommodating change in the LT/LP and its artifacts of practice?

3. How are your approaches to validation and change mutually complementary or in tension?

Defining a learning trajectory:

It is a multi-dimensional theoretical tool focused on a mathematical (STEM) topic that is specific within a curricular guideline. It will coordinate leveled descriptions of actions and verbal patterns, with corresponding hypotheses about mental actions on objects to explain the observable actions and verbal patterns, with corresponding descriptions of relevant misconceptions and instructional activity to identify and to promote growth to the next level. It is based in empirical observation, through immediate research and through the accumulation of findings in extant research.

<table>
<thead>
<tr>
<th>Name of level: brief identifiers for Developmental Progression</th>
<th>Mental actions on objects</th>
<th>Partial knowledge or misconceptions</th>
<th>Appropriate instructional tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>(observable traits)</td>
<td>(inferred thinking, schemes and patterns of reasoning)</td>
<td>(assessment items to fit specific mental actions)</td>
<td>(instructional moves, activities, prompts and tasks to fit specific mental actions)</td>
</tr>
</tbody>
</table>

What have we done to validate, and field test, and implement our LTs?

Five Recommendations for critical review of an existing LT:

1. Incorporate a tension between cross-sectional assessment using IRT analysis (cross-sectional data, over multiple years of development (i.e., grades 1-5, or ages 12 to 15) as complemented by a longitudinal study (i.e., design experiments on learning with the same age span);

2. Assert specific time boundaries as predictions, based on the LT, so that children would be expected to achieve level changes, with
boundaries for experimental variation and plausible methodology so that the assertions can be confirmed or disconfirmed;

(3) Use externally valid terminology within the LT level descriptors and compare against an external, independent LT on the same topic; (blind review of these comparisons should be attempted)

(4) Coordinate the LT with other content domains that may be pertinent (e.g., estimation, geometry, proportional reasoning for a LT characterizing a mathematical domain) to see the “internal validity” of the claims;

(5) Check the LT sequence for reversals of order, omissions or extraneous levels, inconsistent grain-size of descriptions, or inconsistent terminology (progress variables must be coherent). Look for obstacles to incorporation of prior levels in rising levels.

Example of comparing two LTs, (3), (Barrett & Battista, 2014):